**E**stimation **A**nd **C**onfidence **I**ntervals

**Background**

In quality control processes, especially when dealing with high-value items, destructive sampling is a necessary but costly method to ensure product quality. The test to determine whether an item meets the quality standards destroys the item, leading to the requirement of small sample sizes due to cost constraints.

**Scenario**

A manufacturer of print-heads for personal computers is interested in estimating the mean durability of their print-heads in terms of the number of characters printed before failure. To assess this, the manufacturer conducts a study on a small sample of print-heads due to the destructive nature of the testing process.

**Data**

A total of 15 print-heads were randomly selected and tested until failure. The durability of each print-head (in millions of characters) was recorded as follows:

1.13, 1.55, 1.43, 0.92, 1.25, 1.36, 1.32, 0.85, 1.07, 1.48, 1.20, 1.33, 1.18, 1.22, 1.29

**Assignment Tasks**

**a. Build 99% Confidence Interval Using Sample Standard Deviation**

Assuming the sample is representative of the population, construct a 99% confidence interval for the mean number of characters printed before the print-head fails using the sample standard deviation. Explain the steps you take and the rationale behind using the t-distribution for this task.

**b. Build 99% Confidence Interval Using Known Population Standard Deviation**

If it were known that the population standard deviation is 0.2 million characters, construct a 99% confidence interval for the mean number of characters printed before failure.

**Tasks Solution**

Answer) a.) 99% Confidence Interval using Sample Standard Deviation:

To construct a 99% confidence interval for mean number of characters printed before the print-head fails using the sample standard deviation, we will do the following steps:

Given Data:

Number of observations (n) =15

Durability of print-heads (in millions of characters):

1.13, 1.55, 1.43, 0.92, 1.25, 1.36, 1.32, 0.85, 1.07, 1.48, 1.20, 1.33, 1.18, 1.22, 1.29

Now we will calculate Sample Mean (𝑥) and Sample Standard Deviation (s) by the formula:

Sample Mean (𝑥) = 1.3233 million characters

Standard Deviation (s) = 0.0966 million characters

Now, we will calculate the t-score (t-value) for 99% confidence level with degrees of freedom (df) = n - 1:

For a 99% confidence level and df = 14, the t-score (t-value) is approximately 2.977.

Now we will calculate Margin of Error (ME):

ME = 0.0744 million characters

Now construct the Confidence Interval:

Confidence Interval = 1.3233 ± 0.0744

Confidence Interval = (1.2489, 1.3977) million characters

Therefore, the 99% confidence interval for the mean number of characters printed before the print-head fails is approximately (1.2489 million characters, 1.3977 million characters).

Rationale for Using the t-Distribution:

We use the t-distribution because the sample size (n = 15) is small, and the population standard deviation is unknown. The t-distribution accounts for the greater uncertainty in estimating the population mean when working with smaller sample sizes. Using the t-distribution provides a more accurate estimation of the confidence interval compared to using the normal distribution, especially when dealing with small samples.

b.) 99% Confidence Interval Using Known Population Standard Deviation:

If we know that the population standard deviation is 0.2 million characters, we can use the z-distribution instead of the t-distribution because we have information about the population standard deviation.

The formula for the confidence interval in this case is:

Confidence interval = x ± 𝑍 × Population Standard Deviation/​

Given:

Population Standard Deviation (𝜎) = 0.2 million characters

Sample Mean (x) = 1.3233 million characters

Sample Size (n) = 15

Z-score for 99% confidence level = 2.576

We get Confidence Interval by putting the values in the formula = 1.3233 0.1329

Confidence Interval = (1.1904, 1.4562) million characters

Therefore, the 99% confidence interval for the mean number of characters printed before failure, using the known population standard deviation, is approximately (1.1904, 1.4562) million characters.